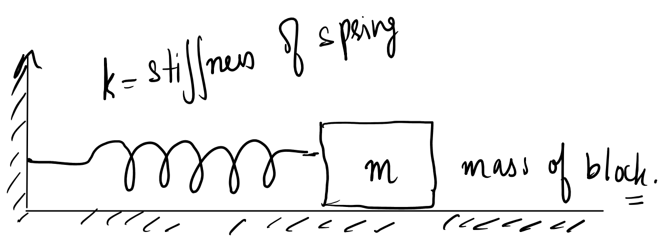
ENERGY IN SIMPLE HARMONIC MOTION (SHM)

As we know, the displacement of particle at an instant in SHM is given by:

$$x(t) = A\sin(\omega t + \phi)$$

Velocity, $$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left(A\sin(\omega t+\phi)\right) = A\omega\cos(\omega t+\phi)$$

acceleration, $$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left(A\omega\cos(\omega t+\phi)\right) = -A\omega^2\sin(\omega t+\phi)$$



$$\omega = \sqrt{\frac{k}{m}}$$

Now, Total mechanical energy of particle in SHM is the sum of potential energy & kinetic energy.

$$\boxed{TE = PE + KE}$$

$$PE(t) = \frac{1}{2}k\left(x(t)\right)^2 = \frac{1}{2}k\left(A\sin(\omega t+\phi)\right)^2 = \frac{1}{2}kA^2\sin^2(\omega t+\phi)$$

$$KE(t) = \frac{1}{2}m\left(v(t)\right)^2 = \frac{1}{2}m\left(A\omega\cos(\omega t+\phi)\right)^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t+\phi)$$

also $\omega^2 = \frac{k}{m}$

$$\therefore \quad KE(t) = \frac{1}{2}\cancel{m}A^2 \times \frac{k}{\cancel{m}}\cos^2(\omega t+\phi) = \frac{1}{2}kA^2\cos^2(\omega t+\phi)$$

Now, $$TE(t) = PE(t) + KE(t)$$

$$= \frac{1}{2}kA^2\sin^2(\omega t+\phi) + \frac{1}{2}kA^2\cos^2(\omega t+\phi)$$

$$= \frac{1}{2}kA^2\left(\sin^2(\omega t+\phi) + \cos^2(\omega t+\phi)\right)$$

$$\frac{1}{2}kA^2\left(\underbrace{\sin^2(\omega t+\phi)+\cos^2(\omega t+\phi)}_{}\right) = 1$$

$$\boxed{TE = \frac{1}{2}kA^2}$$

amplitude of the SHM